

AUBG, in collaboration with the Institute of Mathematics and Informatics (IMI) at the Bulgarian Academy of Sciences (BAS) and Todorov Foundation <https://todorovfoundation.org/> organizes

## Quantum Physics Symmetries: Recollections and Emergent Horizons

Satellite Workshop  
*in memory of Ivan Todorov*

Elieff Center-AUBG, Sofia  
31 May- 1 June, 2026

### Abstracts

#### John Baez

*The Standard Model gauge group from octonions*

Ivan Todorov and Michel Dubois-Violette showed that the Standard Model gauge group can be constructed using the exceptional Jordan algebra, consisting of  $3 \times 3$  self-adjoint matrices of octonions. After an introduction to the physics of Jordan algebras, we ponder the meaning of their construction. Conjecturally, it implies that the Standard Model gauge group consists of the symmetries of an octonionic qutrit that restrict to symmetries of an octonionic qubit, a complex qutrit, and a complex qubit. We explain precisely what this means, and what remains to be proved.

#### David Broadhurst

*Zeta values and Lambert series in quantum field theory*

Ivan Todorov was a much valued source of encouragement, advice and historical perspective during my work with Dirk Kreimer on zeta-valued counterterms in renormalized quantum field theory. I shall recount a result that he highlighted in his 2013 paper with Nikolay Nikolov and Raymond Stora on renormalization in configuration space. Since Ivan was a fine student of the history of ideas, I shall explain how a transformation formula for Lambert series lay hidden in Ramanujan's notebooks, for 50 years. It relates the appearance of zeta values in evaluations of massive Feynman integrals at small and large momenta. Recently, Daniele Dorigoni and I have generalized this result, finding applications to topological string theory at small and large coupling.

#### Keremcan Doğan

*Exceptional Drinfel'd Algebroids and Rackoids*

String and M-theories dictate new symmetry notions that are absent in point-particle theories. The generalized geometry program extends usual differential geometry in a suitable manner to explain one class of these new symmetries, known as T-duality. In particular, Poisson-Lie T-duality can be understood as arising from different decompositions of the Drinfel'd double of a Lie bialgebra, which is itself a Lie algebra. Extending this to the algebroid setting leads to Drinfel'd doubles of Lie bialgebroids, which are Courant algebroids. In order to explain another class of new symmetries, called U-duality, one needs to further extend these notions. In one of our recent works, we extended Lie bialgebroids and their Drinfel'd doubles to a set-up in which the vector bundles

are not dual in the usual sense, and we introduced bialgebroids and their Drinfel'd doubles via a calculus framework on algebroids. In this talk, we use this framework to introduce and construct a specific type of algebroid, which we call exceptional Drinfel'd algebroids. We prove that these are algebroid versions of exceptional Drinfel'd algebras, which have recently been defined in the physics literature in order to extend the Lie bialgebra/T-duality relation to the U-duality case; hence the name. We provide a mathematically rigorous framework to describe these algebras and their algebroid versions in a frame-independent manner, where we use Nambu-Poisson structures and their certain generalizations. Moreover, we introduce exceptional Drinfel'd rackoids, which are global versions of exceptional Drinfel'd algebroids, analogous to the relation between a Lie group and its Lie algebra. As examples, we focus on the  $SL(5)$  and  $E_{6(6)}$  cases; for the latter we also use another extension called proto bialgebroids, where  $H$ - and  $R$ -fluxes are present.

## Mahmut Elbistan

### *Symmetries of Exact Plane Gravitational Waves*

It is well known that the Killing vectors of exact plane GWs are subject to a matrix Sturm-Liouville equation which does not admit a generic solution. There are only a few known exact solutions. Based on one of them, we will argue the possibility for finding Killing tensors of exact plane GWs.

## Vesselin Filev

### *Meson spectra from global Anti-de Sitter space*

In this talk, I discuss probe D7- and D5-branes in global Anti-de Sitter space, dual to flavored  $\mathcal{N} = 4$  theory on  $\mathbb{R} \times S^3$ . These branes undergo a geometrical phase transition tuned by the bare quark mass. By analyzing meson spectra from brane fluctuations, we track this transition numerically at finite mass, and analytically match supergravity with free field theory at zero mass. Ultimately, these results show that mesons cease pair-production once their zero-point energy exceeds their binding energy.

## Kiril Hristov

### *Supersymmetric partition functions from equivariant characteristic classes*

We introduce the equivariant volume, along with the associated equivariant intersection numbers and characteristic classes, for non-compact toric Calabi-Yau manifolds. We then demonstrate, using two specific string theory examples, that supersymmetric partition functions can be expressed in terms of this equivariant data.

## Richard Kerner

### *The Lorentz covariance from discrete $\mathbb{Z}_2$ and $\mathbb{Z}_3$ symmetries*

We argue that the deep origin of the Lorentz covariance resides in symmetries acting on the quantum level. The anticommutation relations resulting from Pauli's exclusion principle engender the  $SL(2, \mathbb{C})$  symmetry in the Hilbert space of two-fermion states, while cubic ternary generalization of the exclusion principle leads to a  $\mathbb{Z}_3$  covering of the Lorentz group also engenders the  $SU(3)$  color symmetry. We show how the  $\mathbb{Z}_3$ -Lorentz symmetry intertwined with the  $SU(3)$  is implemented in a generalized Dirac equation for colored quarks.

## Ognyan Kounchev

### *Harmonic Analysis on Dirac-Klein Quadric*

The Klein-Dirac quadrics may be obtained by means of a proper complexification of the multidimensional ball. As a generalization one may consider a complexification of the multidimensional annulus. An interesting and non-trivial Harmonic analysis arises which is of interest in Physics.

## Alessio Marrani

### *Time as the Simplest Jordan Algebra*

Time reparametrization in conformal quantum mechanics leads to a hidden dynamical  $Sp(2, \mathbb{R})$ -symmetry. Superization and subsequent extensions yield a hierarchy of superconformal extensions,

$$Sp(2, \mathbb{R}) \subset OSp(1|2) \subset OSp(2|2) \subset SU(1, 1|2),$$

which are known to describe the radial dynamics of a superparticle near the event horizon of a Reissner-Nordstrom black hole, as well as of a charged spin-1/2 particle in a magnetic monopole background. We put forward the interpretation of the above chain of embeddings of superconformal extensions of the time reparametrization group  $Sp(2, \mathbb{R})$  through the lens of the Tits-Kantor-Koecher construction for the tiny Kaplansky algebra, which, as from Kac's classification, is the smallest simple Jordan superalgebra (its bosonic part being coordinatized by the time variable). Thus, this perspective regards the time as a Jordan algebra, provides a unifying algebraic framework for superconformal quantum mechanics in external gauge/gravity fields.

## Mustafa Mullahasanoğlu

### *Superconformal Index of Quiver Mechanics with $D(2, 1; \alpha)$ Symmetry*

In this talk, I will briefly introduce the superconformal index and quiver quantum mechanics. We will focus on superconformal quantum mechanics models with  $D(2, 1; \alpha)$  symmetry, which describes the scaling regime of the QQM Coulomb branch. I then explain how to define a refined superconformal index that counts protected short multiplets of  $D(2, 1; \alpha)$ . I will discuss key properties of the index and illustrate them with simple examples.

## Federica Muscolino

### *2T Physics via Freudenthal*

The two time (2T) physics was introduced at the beginning of the nineties by Itzhak Bars. In this paradigm, different (relativistic or non-relativistic) physical systems can be obtained from an *extended* phase space (EPS), with additional time-like and space-like dimensions, by performing suitable gauge fixings, involving the generators of the symplectic symmetry  $Sp(2, \mathbb{R})$  of the EPS itself. In this talk, I propose a method to formalize different gauge fixing associating the EPS properties with reduced Freudenthal triple systems (FTS) constructed over a cubic Jordan algebra. Indeed, the EPS can be endowed with the structure of a FTS over a semi-simple cubic Jordan algebra (called Lorentzian spin factor), with the isometry group of the EPS being isomorphic to the automorphism group of the FTS. Such a non-compact real group has a non-transitive action on the EPS itself, inducing a *stratification* into orbits. Each orbit is defined by classes of algebroid differential constraints on a unique invariant polynomial of degree four. The aforementioned  $Sp(2, \mathbb{R})$  gauge fixing localizes *all* physical systems so far obtained in Bars' theory only onto a



class consisting of two isomorphic orbits of the FTS. Finally, I speculate on the physical meaning of the other orbits, as well as on a method to find other, new and more general gauge fixing within the 2T physics.

## **Misha Shkolnikov**

### *Three scales of Self-Organized Criticality*

The original experiment of Bak, Tang and Wiesenfeld, providing the prototypical example of Self-Organized Criticality, is as a simple Markov chain, having a finite number of recurrent states and operating on a bounded region of a lattice subject to a simple local rule. About ten years ago, jointly with Nikita Kalinin, we have established that a finite truncation of this chain in the high density regime admits a scaling limit effectively described by tropical geometry where observable quantities are now upgraded from discrete to piecewise linear. Moreover, the local symmetries when passing from the discrete to piecewise linear level extend from a finite group to an infinite, although still discrete, group. In a bigger collaboration, we have initiated the study of an analogue of the original BTW Markov chain on this larger scale, where every state is transient and the dynamics builds in complexity indefinitely. This allows to pass to the secondary scaling limit, which presents a new emergence of now continuous observables and their continuous group of symmetries. In my talk, I will describe the three scales of SOC, the ascensions from micro- to meso-, and from meso- to macro- scales, the corresponding mechanisms of gradual symmetry emergence, as well as the partial top-down causation manifesting in the ability to draw concrete statistical predictions at the lattice level from a PDE solution. Based on a work in progress with Higinio Serrano, Ernesto Lupercio, Alexander Varypaev and Nikita Kalinin.

## **Valdemar Tsanov**

### *Quantum entanglement, projective geometry, and equivariant stratifications of the state space*

If a quantum system is modelled on a finite dimensional Hilbert space with an irreducible symmetry group, e.g. many qubits, bosons, or fermions, many of its properties can be interpreted in terms of momentum maps, or in terms of projective geometry related to the variety of coherent states, its secant varieties, osculating spaces, and other classical constructions. In this talk I will discuss some such interpretations, and generic versus special phenomena for various systems.

## **Cem Yetişmişoğlu**

### *Odd Dimensional Generalizations of Supersymplectic Manifolds and Discrete Dynamical Systems*

Supersymplectic manifolds provide a natural geometrical framework to study systems with both bosonic and fermionic degrees of freedom. Moreover, by integrating in a time direction, one can generalize supersymplectic manifolds to odd dimensional counterparts which incorporate dynamics as part of the geometry. In this geometrical generalized setting, we will be presenting a measurement theory for discrete systems and their dynamics.